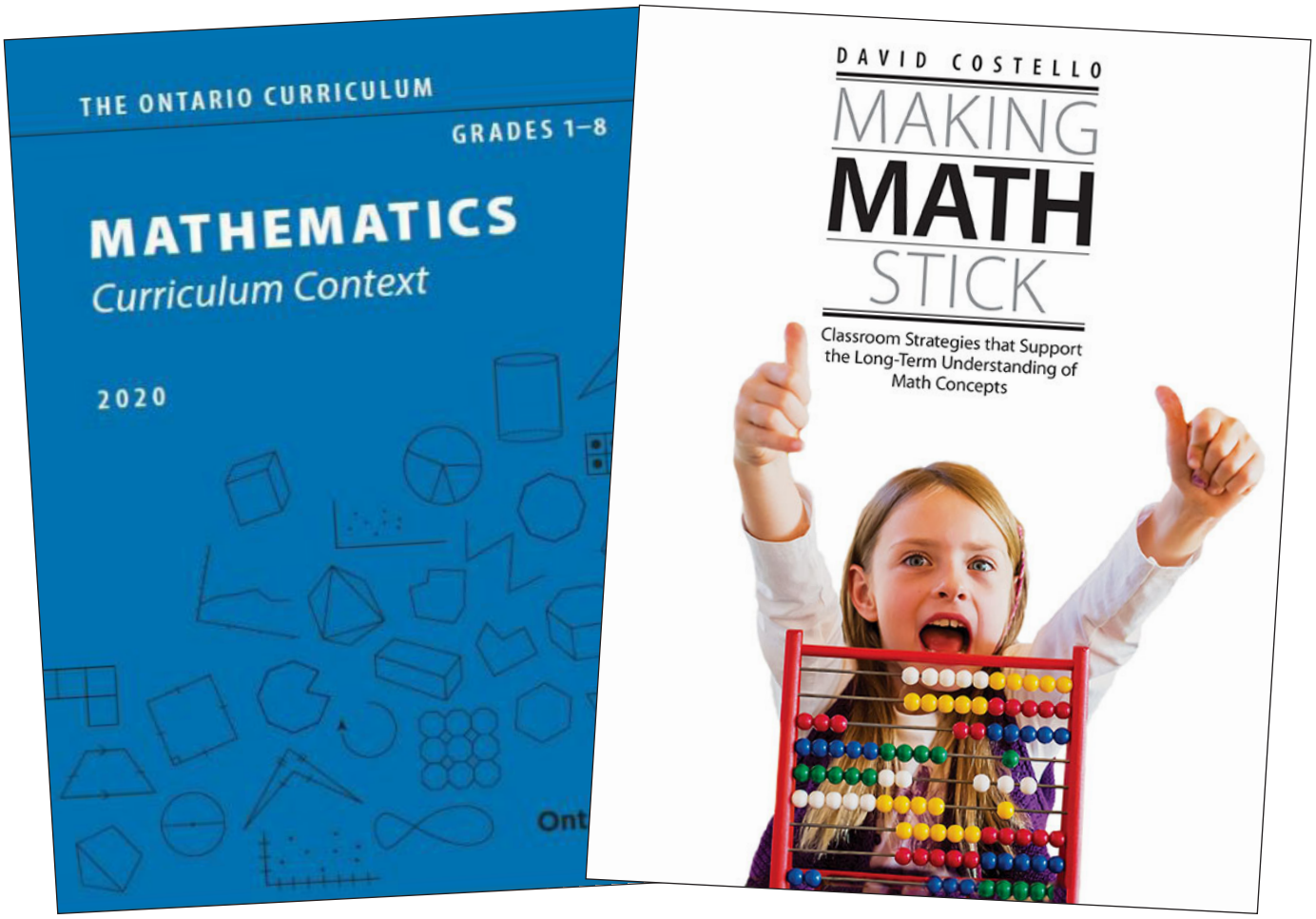
DAVID COSTELLO

THE INSIDER'S GUIDE TO THE ONTARIO MATH CURRICULUM



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Contents

Introduction

The Ontario Ministry of Education revised the Grades 1–8 mathematics curriculum in 2020. When comparing the previous mathematics curriculum (2005) and the revised mathematics curriculum (2020), there are notable differences. The revised mathematics curriculum (2020):

- Focuses on fundamental mathematics concepts and skills. In addition to this, there is significant emphasis placed on making connections between related math concepts, between mathematics and other disciplines, and between mathematics and everyday life.
- Supports new learning about mathematical modelling, coding, and financial literacy.
- Stresses the importance of social-emotional learning. The curriculum is structured to highlight necessary skills in students having a positive attitude towards mathematics and being able to become capable and confident math learners.

High-Impact Instructional Practices in Mathematics

To support educators in facilitating the revised mathematics curriculum (2020), the Ontario Ministry of Education released a supporting document that would highlight instructional practices providing a high-impact on student learning. This document, *High-Impact Instructional Practices in Mathematics,* provides an overview of instructional practices that are to be used in collaboration with the revised mathematics curriculum (2020).

Making Math Stick

Making Math Stick: Classroom Strategies that Support the Long-Term Understanding of Math Concepts closely aligns with Ontario's revised mathematics curriculum (2020). *Making Math Stick* broadens the perspective of learning to include three stages:

- **Encoding** means getting knowledge into our heads.
- **Consolidation** means assigning meaning to that knowledge, which gets it into our memory. It is through this process that connections are established between recently encountered knowledge and knowledge already stored in long-term memory.
- **Retrieval** means getting knowledge out of our heads. Retrieval is the process of reaching back and bringing something we previously learned into mind.

As referenced throughout Ontario's revised mathematics curriculum (2020), it is no longer sufficient to focus instruction on getting knowledge into the heads of students. *Making Math Stick* provides educators with numerous instructional and learning strategies that will assist them in facilitating Ontario's revised mathematics curriculum (2020). *Making Math Stick* is about strategies that support teachers and students in spacing and mixing concepts throughout the year so that students can remember and apply previously learned math concepts. It is filled with strategies that are manageable and sustainable, and that make a difference. Working smarter, not harder. The focus, within *Making Math Stick*, is **students thinking mathematically** opposed to **students doing math**.

In the pages that follow, I will:

- examine each of the six strands within Ontario's revised mathematics curriculum (2020) and highlight any changes in relation to the previous mathematics curriculum (2005);
- share one learning strategy from *Making Math Stick* that students can apply to support understanding (across the grade levels) within the particular strand; and
- share one instructional strategy from *Making Math Stick* that educators can leverage to support instruction (across the grade levels) within the particular strand;
- highlight the instructional practice that this applies to as documented within the Ontario Ministry of Education's supporting document *High-Impact Instructional Practices in Mathematics*.

STRAND A

Social-Emotional Learning Skills in Mathematics and Mathematical Processes

Strand A	Strand B	Strand C	Strand D	Strand E	Strand F
Social-Emotional Learning Skills in Mathematics and Mathematical Processes	Number	Algebra	Data	Spatial Sense	Financial Literacy

This strand focuses on students' development and application of social-emotional learning skills to support their learning of math concepts and skills, foster their overall well-being and ability to learn, and help them build resilience and thrive as math learners. Social-emotional learning skills support students in understanding mathematical concepts and in applying the mathematical processes that are key to learning and doing mathematics. They help all students develop confidence, cope with challenges, and think critically. This in turn enables them to improve and demonstrate mathematics knowledge, concepts, and skills in a variety of situations. Social-emotional learning skills help every student develop a positive identity as a capable "math learner".

Students' application of social-emotional learning skills and mathematical processes must be assessed and evaluated as a part of their achievement of the overall expectations in each of the strands for every grade. The following are the social-emotional learning skills and mathematical processes:

Social-Emotional Learning Skills	Mathematical Processes
• Identification and Management of Emotions • Stress Management and Coping • Positive Motivation and Perseverance • Healthy Relationship Skills • Self-Awareness and Sense of Identity • Critical and Creative Thinking	• Problem Solving • Reasoning and Proving • Reflecting • Connecting • Communicating • Representing • Selecting Tools and Strategies

Key Changes in the Revised Mathematics Curriculum (2020)

Social-emotional learning skills were not included in the previous math curriculum (2005).

Learning Strategy Category

Using Awareness Strategies to Improve Learning: Helping students consciously improve their learning process:

- Prioritize aspects of a problem
- Summarize a problem-solving experience
- Reflect on a problem-solving experience
- Use a learning journal to record key takeaways
- Record their next steps

The following are examples of primary, elementary and intermediate students reflecting, summarizing, and recording their experiences:,

PRIMARY EXAMPLE: STUDENT WORK

Reflecting on what was successful

A teacher wanted to help a discouraged primary student. She suggested that the student refocus her reflections on what went well during math sessions. Doing this helped the student focus on the positive. She began to use what went well to solve other problems—and it helped!

ELEMENTARY EXAMPLE: STUDENT WORK

Summarizing favored strategies

A student looked through multiple samples of their own work and listed the strategies they typically applied to solve problems of a particular type. Compiling this summary made them more aware of the strategies they gravitated to and strengthened their recall of these strategies.

INTERMEDIATE EXAMPLE: STUDENT WORK

Recording success with rational numbers

An intermediate student records the path she took to success ordering rational numbers:

Rational numbers can be hard to order. Lots of fractions, integers, and decimals. It took a while, but I can order all of them now. I was having trouble with fractions, but the more I practiced the better I got.

Instructional Strategy Category

Rethinking Year Plans: Planning the year to interrupt the forgetting process:

- Creating a **Spiralled Curriculum Year Plan**
- Creating a **Key Concepts Year Plan**

The following are examples of how elementary and intermediate teachers transitioned to a spiralled curriculum year plan and a key concepts year plan:

ELEMENTARY EXAMPLE: A TEACHER EXPLAINS

Designing a spiralled curriculum year plan

Looking through the grade level curriculum, there were many outcomes I had to consider. I wanted to group outcomes together in a way that would be meaningful for students, that would highlight the connections amongst the outcomes, and that would allow me to cycle through them throughout the year while increasing the complexity during each cycle.

I decided to group outcomes according to the big ideas in math. Each big idea served as an umbrella topic for multiple curriculum outcomes from multiple strands.

For each big idea, I examined the curriculum to identify the outcomes that fit well. While some outcomes could go under multiple big ideas, I decided to group the individual outcomes that had the strongest connections. But I made a point of looking for outcomes in multiple strands. Within each big idea, it was interesting to see various strands addressed. Through this process, I was making connections across the curriculum. Overall, I ended up with eight big ideas.

Next, I chunked up each big idea into three sections. The average time that I needed to spend on each section was approximately two weeks (with some sections requiring less time while others required a bit more, depending on the complexity of the idea and the learning needs of students). The key was to divide the teaching of each big idea into three sections, and cycle through the sections during the year.

The first section was more introductory, skimming the surface of concepts. After I cycled through the first section of all the big ideas, I then addressed the second sections of the big ideas. The second section went a bit more in-depth. After I addressed all the second sections, I then moved into the third sections. The third sections went deepest. It was like burrowing into the big ideas as the year progressed.

I was cognizant of not leaving students with the impression that they were just repeating concepts as they moved through each cycle. Instead, as students engaged with the same big idea three times during the year, they encountered different contexts each time. It was not about repeating the big ideas so much as it was about expanding the learning experiences of students throughout the year.

INTERMEDIATE EXAMPLE: A TEACHER EXPLAINS

Transitioning to a key concepts year plan

It just makes so much sense. That is what I thought when I first heard about frontloading the year plan with key concepts.

Looking back, why would I wait until late winter or early spring to introduce a concept that I knew would help students with other concepts and that was critical to student success? If I must be honest, sometimes I would have to rush through these key concepts because I was behind in the year plan by the time they came around in the plan.

Now I spend the first part of the year on the key concepts so that students have plenty of time to work with them and I don't have to rush through them. Plus, I have found that doing these concepts early has helped students with other concepts that come later in the year.

The other thing I noticed was that I can return to some of these key concepts later in the year if students are struggling with them. My year plan now gives the students a lot more time to work with and learn the concepts.

When I see students are having difficulty with key concepts in the spring, I will give a few assessments and then provide corrective feedback. I wouldn't be able to do that with my old year plan because I didn't have the time to go back, as it was much later in the year when these concepts were introduced.

Correlation to Ontario Ministry of Education's supporting document *High-Impact Instructional Practices in Mathematics*

Learning Goals, Success Criteria, and Descriptive Feedback

Students do much better when they know what to do and what the intention of the learning is. Clearly articulated learning goals and success criteria provide students with the target and necessary steps, while descriptive feedback provides students with their next steps to achieve the target.

Math Conversations

Conversations focused on mathematics support student learning. Whether students listen or talk with peers in partners, small groups, or the whole class, students are engaged in a learning experience that will enable them to compare and contrast their understanding, and have opportunities to explore mathematical concepts from differing perspectives.

STRAND B

Number

Strand A	**Strand B**	Strand C	Strand D	Strand E	Strand F
Social-Emotional Learning Skills in Mathematics and Mathematical Processes	**Number**	Algebra	Data	Spatial Sense	Financial Literacy

Understanding how numbers work is foundational to many aspects of mathematics. In the Number strand, as students progress through Grades 1 to 8, they learn about different types of numbers and how those numbers behave when various operations are applied to them.

A vital aspect of number work in the elementary grades is to build what is often called number sense, where students develop the ability to flexibly relate numbers and relate computations. Students who have developed number sense regularly use number relationships to make sense of calculations and to assess the reasonableness of numbers used to describe situations, for example, in the media.

Students learn to count effectively and then become fluent with math facts so they can perform calculations efficiently and accurately, whether mentally or by using algorithms on paper. This strand is built on the belief that it is important to develop automaticity, which is the ability to use mathematical skills or perform mathematical procedures with little or no mental effort. Automaticity with math facts enables students to engage in critical thinking and problem solving.

Most students learn math facts gradually over years. Mastery comes with practice, and practice helps to build fluency and depth. Students draw on their ability to apply math facts as they manipulate algebraic expressions, equations, and inequalities. Mental math skills involve the ability to perform mathematical calculations without relying on pencil and paper. They enable students to estimate answers to calculations, and to work accurately and efficiently on everyday problems and judge the reasonableness of answers that they have arrived at through calculation. To develop effective mental math strategies, all students need to have strong skills in number sense and a solid conceptual understanding of operations.

Key Changes in the Revised Mathematics Curriculum (2020)

Recalling multiplication and division facts.

Learning Strategy Category

Learning by Picturing It: Helping students create images to explore problems and strengthen memory:

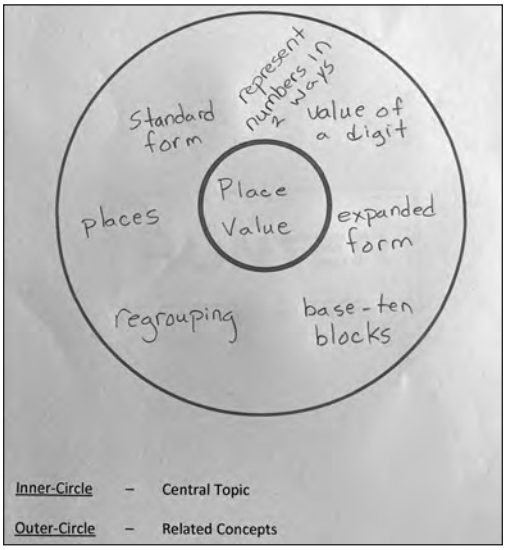
- **Visualize** a problem to "see" it
- **Free sketch** to explore a problem
- Create **concept maps** to find connections and relationships
- Create **graphic organizers** to organize thinking

The following are examples of primary, elementary, and intermediate students using a graphic organizer, creating a concept map, and visualizing:

PRIMARY EXAMPLE: STUDENT WORK

Using a circle graphic organizer to explore place value

The following is a circles graphic organizer on place value completed by a primary student.



ELEMENTARY EXAMPLE: STUDENT WORK

Visualizing base-ten blocks to help with multiplication

When working with multiplication of two-digit by one-digit numbers, a student attempted to use an array to help with understanding. For this particular student, though, the use of an array did not aid in understanding the problem or in reaching the solution. What eventually helped him understand the process of multiplying was visualizing base-ten blocks.

INTERMEDIATE EXAMPLE: STUDENT WORK

Visualizing fraction strips to subtract fractions

Subtraction of fractions with unlike denominators can be challenging. An intermediate student shared how she would picture fraction strips in her mind as she was subtracting. She pictured removing just the portion of the fraction needed. This enabled her to visualize the difference. For example, in $\frac{9}{10} - \frac{2}{5}$, the student visualized $\frac{9}{10}$ and then removed $\frac{2}{5}$ from that visualization. This took two steps: first she removed $\frac{2}{10}$ and then another $\frac{2}{10}$ (for a total of $\frac{4}{10}$). This left the difference of $\frac{5}{10}$ remaining. Maintaining a visual as she worked helped her understand the process of subtraction.

Instructional Strategy Category

Rethinking Review: Planning spaced and mixed reviews to interrupt the forgetting process:

- Creating Daily Cumulative Reviews

The following are examples of how primary, elementary, and intermediate teachers implemented a daily cumulative review:

PRIMARY EXAMPLE: A TEACHER'S EXPERIENCE

Mixing concepts for a week of daily cumulative review

In this example, the teacher identified place value, perimeter, subtraction, and equations as the concepts she wanted to emphasize during the review. Notice how the concepts are arranged in a different order from day to day. (The teacher would continue focusing on these four concepts for several weeks, and then would revisit them occasionally during the rest of the schoolyear.)

Monday	1. What is the value of the underlined digit? 4<u>9</u>2 2. Find the perimeter of a square with a side length of 7 m. 3. Solve: 568 – 197 4. Find the value of ? 13 = 6 + ?
Tuesday	1. What is the value of the underlined digit? 80<u>9</u> 2. Solve: 701 – 222 3. Find the perimeter of a rectangle with a length of 5 m and width of 3 m. 4. Find the value of ? 12 = ? – 4
Wednesday	1. Solve: 984 – 421 2. Find the perimeter of a five-sided shape with each side length being 8 m. 3. Find the value of ? ? = 5 + 9 4. What is the value of the underlined digit? <u>4</u>92
Thursday	1. Find the value of ? 9 – 8 = ? 2. Find the perimeter of a square with a side length of 8 cm. 3. What is the value of the underlined digit? <u>6</u>39 4. Solve: 432 – 224
Friday	1. Solve: 600 – 286 2. Find the perimeter of a rectangle with a length of 5 cm and width of 8 cm. 3. Find the value of ? ? = 7 – 5 4. What is the value of the underlined digit? 7<u>1</u>3

ELEMENTARY EXAMPLE: A TEACHER'S EXPERIENCE

Mixing concepts for a week of daily cumulative review

In this example, the teacher identified fractions, place value, decimals, and multiplication as the concepts she wanted to emphasize during the review. Notice how the concepts are arranged in a different order from day to day. (The teacher would continue focusing on these four concepts for several weeks and then revisit them occasionally thereafter.)

Monday	1. Write the following number in words: 583 839.34 2. Arrange the following in ascending order: 0.532, 0.523, 0.352, 0.325 3. Solve: 68×34 4. Place >, <, or = in the circle: $\frac{5}{6} \bigcirc \frac{7}{8}$
Tuesday	1. Arrange the following in descending order: 8.041, 8.140, 8.104, 8.014 2. Write the following number in words: 76 284.481 3. Place >, <, or = in the circle: $\frac{2}{5} \bigcirc \frac{2}{6}$ 4. Solve: 42×26
Wednesday	1. Solve: 98×98 2. Place >, <, or = in the circle: $\frac{4}{7} \bigcirc \frac{5}{8}$ 3. Arrange the following in ascending order: 7.22, 2.72, 2.27, 7.72 4. Write the following number in words: 900 290.075
Thursday	1. Arrange the following in descending order: 23.873, 23.837, 23.738, 23.783 2. Place >, <, or = in the circle: $\frac{3}{4} \bigcirc \frac{2}{3}$ 3. Write the following number in words: 38.098 4. Solve: 56×47
Friday	1. Write the following number in words: 509.006 2. Solve: 38×25 3. Place >, <, or = in the circle: $\frac{3}{5} \bigcirc \frac{4}{9}$ 4. Arrange the following in ascending order: 63.429, 63.294, 63.942, 63.924

INTERMEDIATE EXAMPLE: A TEACHER EXPLAINS

Building a mixed approach to review

I was frustrated with students forgetting significant math concepts by the end of the year. Each year, I would have to do a week of review before the exam to ensure that students were well prepared and would remember the year's content.

The last few years, I have changed my approach. Now what I do is build opportunities for review throughout the year. I tried to include all concepts, but I narrowed it down to the big things that students need to know and that carry the most weight in the curriculum.

I have six or seven concepts that I include on each review, one or two questions for each concept, and I give it to my class every four to five weeks. What this does is students see the concepts throughout the year and must solve problems for each of the important ones.

In each review, I mix the order of the questions so that students don't get the same concept for several questions in a row. I want them to spend time thinking about the question, deciding what they need to do, and then doing it.

After each review, I go over the questions with the class. Overall, the mixed review takes 15 to 20 minutes every four to five weeks. Not a lot of time but a great value.

I must say that now, at the end of the year, I am not worried about going over everything that we worked with earlier. Instead, students have a better understanding of the concepts and are ready for the exam.

Correlation to Ontario Ministry of Education's supporting document *High-Impact Instructional Practices in Mathematics*

Problem Solving Tasks and Experiences

Problem solving tasks and experiences provide an opportunity for students to engage with mathematical processes. Open-ended tasks provide students with multiple points of entry and exit, thereby making them accessible to students at various stages of readiness and giving more students an opportunity to construct mathematical ideas.

Deliberate Practice

Practice is a necessary component of effective math instruction. Practice is best when it is deliberate, purposeful, and spaced. Regardless of the form of practice, ongoing feedback is crucial, so that students know that they are practicing correctly and that they have practiced sufficiently. Students need to practice metacognition so that learning becomes self-directed.

STRAND C

Algebra

Strand A	Strand B	**Strand C**	Strand D	Strand E	Strand F
Social-Emotional Learning Skills in Mathematics and Mathematical Processes	Number	**Algebra**	Data	Spatial Sense	Financial Literacy

In this strand, students develop algebraic reasoning through working with patterns, variables, expressions, equations, inequalities, coding, and the process of mathematical modelling.

As students progress through the grades, they study a variety of patterns. Students learn to identify regularities in numeric and non-numeric patterns and classify them based on the characteristics of those regularities. They create and translate patterns using various representations. Students determine pattern rules for various patterns to extend them, make near and far predictions, and determine their missing elements. In the primary grades, students focus on understanding which quantities remain the same and which can change in everyday contexts, and on how to establish equality between numerical expressions. In the junior and intermediate grades, students work with variables in algebraic expressions, equations, and inequalities in various contexts.

Key Changes in the Revised Mathematics Curriculum (2020)

No reference to coding in the previous math curriculum (2005).

Coding

As students progress through the grades, their coding experiences also progress, from representing movements on a grid, to solving problems involving optimization, to manipulating models to find which one best fits the data they are working with to make predictions. Coding can be incorporated across all strands and provides students with opportunities to apply and extend their math thinking, reasoning, and communicating. Students in all grades also engage in the process of mathematical modelling.

No reference to the mathematical modelling process in the previous math curriculum (2005).

The Mathematical Modelling Process

Mathematical modelling provides authentic connections to real-life situations. The process starts with ill-defined, often messy real-life problems that may have several different solutions that are all correct. Mathematical modelling requires the modeller to be critical and creative and make choices, assumptions, and decisions. Through this process, they create a mathematical model that describes a situation using mathematical concepts and language, and that can be used to solve a problem or make decisions, and to deepen understanding of mathematical concepts.

The process of mathematical modelling has four key components that are interconnected and applied in an iterative way—students may move between and across, as well as return to, each of the four components as they change conditions to observe new outcomes until the model is ready to be shared and acted upon. While moving through these steps, social-emotional learning skills and mathematical processes are applied as needed.

1. Understand the problem
- What questions need answering?
- What information is needed?
2. Analyze the situation
- What assumptions do I make about the situation?
- What changes; what remains the same?
3. Create a mathematical model
- What representations, tools, technologies, and strategies will help build the model?
- What mathematical knowledge, concepts, and skills might be involved?
4. Analyze and assess the model
- Can this model provide a solution?
- What are alternative models?

Learning Strategy Category

Learning By Writing: Helping students use informal, exploratory writing to help them think through mathematical concepts.

- Paraphrase a problem to help them understand it
- Approach a problem by doing a free write
- Do a free write of 30 words or less
- Use a plus/minus chart to record what they know and don't know

The following are examples of primary, elementary, and intermediate students writing to learn, paraphrasing, and using a plus/minus chart:

PRIMARY EXAMPLE: A STUDENT EXPLAINS

Writing to learn about patterns

We did a lot of work with patterns. That is how we started the year the last two years. I kept getting mixed up with what the repeating part of the pattern was. So I started to write down everything I knew about patterns in point form. I didn't care about periods or uppercase letters because that wasn't the point. The point was to get as much about patterns out of my head as possible. At first, when I did this, I couldn't write much. But every week when I did this I would write more and more. Now, after about a month, I remember a lot more about patterns.

ELEMENTARY EXAMPLE: STUDENT WORK

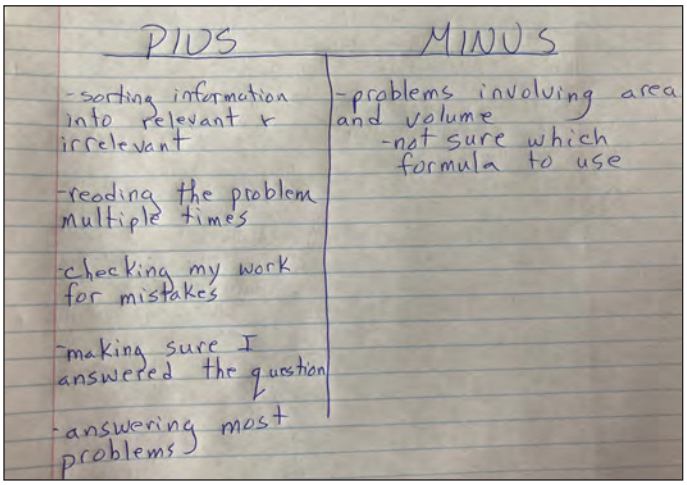
Paraphrasing definitions

Math vocabulary can be challenging for students to understand. An elementary student recorded all the terms and meanings. Then she stated what the term meant in her own words. This process helped her own the learning, and it was a great opportunity to check for understanding and recall.

INTERMEDIATE EXAMPLE: STUDENT WORK

Using a plus/minus chart to assess word-problem skills

An intermediate student explores her thinking when solving word problems.



Instructional Strategy Category

Focusing Instruction on Key Concepts: Prioritizing concepts that best support student learning across mathematics:

- Continually return to key concepts throughout the year

The following is an example of how an elementary teacher approached focusing instruction on key concepts:

ELEMENTARY EXAMPLE

List of key concepts

The following is a list of the key concepts a teacher crafted for an elementary math class.

- Read, write, and compare decimals to thousandths.
- Fluently multiply multi-digit whole numbers using the standard algorithm.
- Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and area models.
- Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
- Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.
- Interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
- Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

Correlation to Ontario Ministry of Education's supporting document *High-Impact Instructional Practices in Mathematics*

Teaching About Problem Solving

Teaching about problem solving makes the implicit explicit. Students are supported in seeing the thinking that problem solving entails. It helps students understand the structure of a problem and reaffirms that problem solving requires perseverance and a growth mindset.

STRAND D

Data

Strand A	Strand B	Strand C	**Strand D**	Strand E	Strand F
Social-Emotional Learning Skills in Mathematics and Mathematical Processes	Number	Algebra	**Data**	Spatial Sense	Financial Literacy

Statistics and probability are highly relevant to real life. The public is bombarded with data through advertising, opinion polls, politics, population trends, and scientific discoveries, to name just a few. Thus, one of the key focuses in this strand is to support students in developing critical thinking skills throughout their development of data literacy, so that they can analyze, synthesize, understand, generate, and use data, both as consumers and producers.

The main purpose for collecting and organizing data is to gather information to answer questions. When students collect and organize data, they have an opportunity to learn more about themselves, their environment, issues in their school or community, and so on. Learning activities should help students understand the processes that are involved in formulating questions, seeking relevant information, and organizing that information in meaningful ways. Involving students in collecting and organizing data allows them to participate in the decision making that is required at different steps of the process.

Students learn the fundamentals of statistics and develop the skills to visualize and critically analyze data, including identifying any possible biases within the data. Starting in the junior grades, students make intentional choices in creating infographics to represent key information about a set of data for a particular audience and to engage in the critical interpretation of data. In addition, students learn how to use data to make compelling arguments about questions of interest.

The learning in this strand also supports students in developing probabilistic reasoning. As students progress through the grades, they begin to understand the relationship between probability and data, and how data is used to make predictions about populations. Students begin to understand and represent these probabilities as fractions, decimals, and percent.

Key Changes in the Revised Mathematics Curriculum (2020)

No reference to reading or creating infographics in previous math curriculum (2005).

Learning Strategy Category

Building a Network of Memory: Helping students elaborate their knowledge of a concept by adding layers of meaning to a memory.

- Create and answer how and why questions
- Explain thinking
- Use dual coding
- Compare and contrast concepts
- Generate concrete examples of abstract knowledge

The following are examples of primary, elementary, and intermediate students explaining thinking and comparing and contrasting concepts:

PRIMARY EXAMPLE: STUDENT WORK

Summarizing a lesson

After a lesson, a primary student listed the main points of the lesson to his peers in a small group during consolidation. This act of summarizing gave the student a chance to recall the highlights of the lesson and to put these together in a concise and coherent manner.

ELEMENTARY EXAMPLE: STUDENT WORK

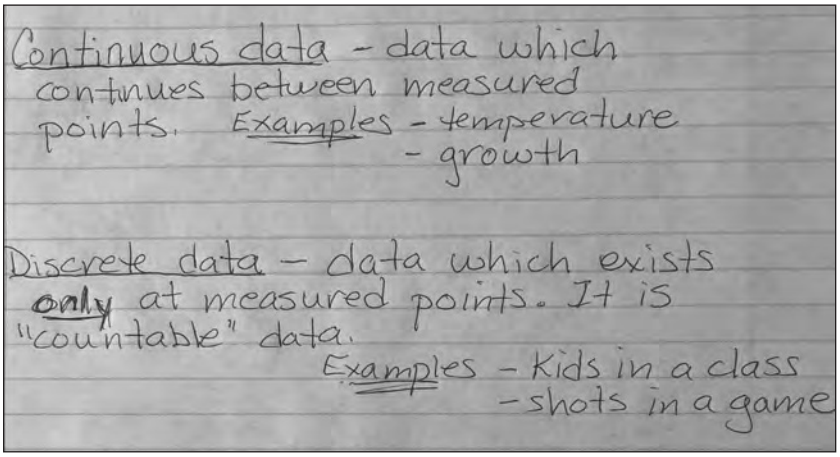
Predicting based on theoretical probability

A student was working on probability and had to make a prediction of what color tile he would pick out of a mystery bag. Equal numbers of red, green, and yellow tiles had been placed in the bag—6 of each. After 1 red, 5 green, and 3 yellow tiles were drawn, the student made a prediction. He said red because he knew there were more red tiles left in the bag than the other two colors added together.

INTERMEDIATE EXAMPLE: STUDENT WORK

Writing definitions of types of data to highlight similarities and differences

A student wrote these descriptions of continuous and discrete data to show how they are similar and different.



Instructional Strategy Category

Adjusting Common Classroom Tools and Strategies: Applying a retrieval practice approach to common instructional tools and strategies:

- Using exit tickets to spark recall
- Teaching with then-now-later grids
- Posting choice boards
- Use think-pair-share for review
- Alternating figure-it-out and here-it-is

The following are examples of how primary, elementary, and intermediate teachers adjusted exit tickets, choice boards, and then-now-later grids:

PRIMARY EXAMPLE: A TEACHER EXPLAINS

Getting over students' initial wariness

I had heard about supporting children in being able to recall previous learning and I wanted to do this in my classroom. However, I was overwhelmed at making a change that I thought would take a lot of preparation and time to do. I was wrong. It wasn't anything big, just more strategic in how I approached my teaching.

I used exit tickets a lot in my classroom. So what I did was start asking students questions about concepts they had encountered earlier in the year instead of that day. The new approach surprised students at first, and they didn't do that well. But over a few weeks (I gave one exit ticket a week), students started to do very well recalling things.

I now use the strategy in my morning message, where I bring up a math concept that they haven't seen in a while, and it is going great.

It really is a small change but makes such a big, big difference.

Elementary Choice Board

Working with Fractions

Represent

Draw a picture to represent the following fractions:

a. $\frac{3}{4}$

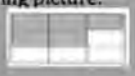
b. $\frac{1}{3}$

c. $\frac{7}{10}$

d. $\frac{5}{8}$

Label

Write a fraction for the following picture:

a. 

b. 

c. 

Compare

Place >, < or = between the following:

a. $\frac{1}{4}$ $\frac{1}{5}$

b. $\frac{1}{3}$ $\frac{3}{7}$

c. $\frac{4}{5}$ $\frac{5}{6}$

d. $\frac{6}{8}$ $\frac{3}{4}$

Order

Arrange the following fractions in descending order:

a. $\frac{1}{5}$ $\frac{4}{5}$ $\frac{3}{7}$

b. $\frac{3}{6}$ $\frac{5}{8}$ $\frac{2}{3}$

c. $\frac{3}{6}$ $\frac{3}{5}$ $\frac{3}{8}$

Number Line

Place the following fractions on a number line:

a. $\frac{1}{5}$ $\frac{4}{5}$ $\frac{2}{5}$

b. $\frac{11}{5}$ $\frac{9}{6}$ $\frac{1}{3}$

c. $\frac{3}{4}$ $\frac{9}{12}$ $\frac{13}{7}$

Fractions & Decimals

Express the following fractions as decimals:

a. $\frac{3}{5}$

b. $\frac{1}{4}$

c. $\frac{9}{10}$

d. $\frac{3}{4}$

Fractions & Mixed Numbers

Express the following improper fractions as mixed numbers:

a. $\frac{13}{3}$

b. $\frac{21}{6}$

c. $\frac{9}{4}$

d. $\frac{20}{7}$

Equivalent Fractions

Write 2 equivalent fractions for each of the following:

a. $\frac{5}{6}$

b. $\frac{1}{4}$

c. $\frac{6}{10}$

d. $\frac{2}{4}$

Mixed Numbers & Fractions

Express the following mixed numbers as improper fractions:

a. $2\frac{1}{3}$

b. $1\frac{2}{5}$

c. $3\frac{4}{7}$

d. $4\frac{2}{5}$

INTERMEDIATE EXAMPLE: A TEACHER'S EXPERIENCE

Designing a then-now-later grid for intermediate students

An intermediate teacher had worked on integers earlier in the year and was now focusing instruction on addition and subtraction of fractions. Coming up next in the year plan was multiplication of whole numbers and decimal numbers. The following is an example of one of the then-now-later grids she used during that time.

Solve: (–9) + (–8)	Solve: (+1) + (–5)	Solve: (–4) – (–7)
Solve: $\frac{5}{6} - \frac{2}{3}$	Solve: $\frac{2}{7} + \frac{1}{3}$	Solve: $\frac{5}{8} - \frac{1}{2}$
$5632 \times 1 = 5632$ How does knowing the above equation help solve 5632×0.1?	$64 \times 0.1 = 6.4$ How does knowing the above equation help solve 64×0.001?	$7462 \times 1 = 7462$ How does knowing the above equation help solve 7462×0.001?

Correlation to Ontario Ministry of Education's supporting document *High-Impact Instructional Practices in Mathematics*

Small-Group Instruction

Small-group instruction is targeted instruction that meets the learning needs of specific students at specific times. Working with small and flexible groups, whether the groups are homogenous or heterogeneous, provides the teacher with an opportunity to tailor instruction to the learning needs of a small group of students at a particular time.

STRAND E

Spatial Sense

Strand A	Strand B	Strand C	Strand D	**Strand E**	Strand F
Social-Emotional Learning Skills in Mathematics and Mathematical Processes	Number	Algebra	Data	**Spatial Sense**	Financial Literacy

This strand combines the areas of geometry and measurement to emphasize the relationship between the two areas and to highlight the role of spatial reasoning in underpinning the development of both. Study in this strand provides students with the language and tools to analyze, compare, describe, and navigate the world around them.

In this strand, students analyze the properties of shapes—the elements that define a shape and make it unique—and use these properties to define, compare, and construct shapes and objects, as well as to explore relationships among properties. Students begin with an intuition about their surroundings and the objects in them, and learn to visualize objects from different perspectives. Over time, students develop an increasingly sophisticated understanding of size, shape, location, movement, and change, in both two and three dimensions. They understand and choose appropriate units to estimate, measure, and compare attributes, and they use appropriate tools to make measurements. They apply their understanding of the relationships between shapes and measurement to develop formulas to calculate length, area, volume, and more.

Key Changes in the Revised Mathematics Curriculum (2020)

Minor changes made throughout as compared to the previous math curriculum (2005).

Learning Strategy Category

Learning Through Self-Assessment: Helping students use objective tools to gain a realistic perspective of what they don't know yet and to consolidate what they do know:

- Create their own self-quizzes
- Use flashcards for mixed and spaced review
- Self-monitor

The following is an example of primary, elementary, and intermediate students using self-quizzing, self-monitoring, and flashcards:

PRIMARY EXAMPLE: STUDENT WORK

Predicting whether various 3-D solids roll

Students were provided 3-D solids to sort using one attribute. One student chose to sort based on if the solid would roll on a slope. For each solid, she made a prediction and then tested her prediction by placing the solid on a slope. Each prediction helped her envision what would happen before the actual experiment. The combination of prediction and confirmation strengthened her learning about the attributes of various shapes.

ELEMENTARY EXAMPLE: STUDENT WORK

Self-monitoring work on geometry problems

I'm working with translations, rotations, and reflections…. I'm not sure if my rotations are done right. Should I ask the teacher? No, I want to do it myself.… So, I'll start by putting a question mark beside anything that I'm not sure of and an exclamation mark beside things that I know for sure…. Okay, now I'll go back to the question marks and think about it a little more…. I can make some small changes…. Good, that's right now.

INTERMEDIATE EXAMPLE: STUDENT WORK

Using flashcards to work with math terms and tasks

A student had two colors of flash cards. Green cards had a math term on the front and a definition on the back. The student went through the pile of cards one by one. For each math term, she recalled the definition and then looked at the back of the flashcard to check her response for accuracy.

Blue cards each had a task on the front of the card and a solution on the back. The student worked through the task and then compared her response to the solution on the back of the card.

Instructional Strategy Category

Rethinking Testing: Using frequent testing to spark more frequent retrieval:

- Designing cumulative tests to help students maintain memories
- Designing carry-over tests to maintain learning of difficult concepts

The following are examples of how intermediate teachers crafted a cumulative test and carry-over test:

INTERMEDIATE EXAMPLE: A TEACHER EXPLAINS

Building a cumulative unit test

I always gave tests at the end of a unit. Didn't matter how long the unit was, students would be tested at the end. It would be worth a certain percentage of the course grade.

I always wondered if I was getting what I wanted out of the tests. If a student didn't do well on a test, it didn't change anything because we were onto the next unit. If a student did well on a test, I wasn't sure if they would be able to remember it in the future because we never went back to the concept.

So what I did was start using cumulative tests. I would identify key concepts for the year and then structure my tests to account for any of these that students had worked with. If the first unit was dealing with a key concept, that key concept would be the focus of the first test. Then, when we addressed the next key concept, the test for that unit would focus on the key concept for that unit but also have questions on the previous key concept. This would continue throughout the year so that each test addressing a key concept would have additional questions tagged onto it that targeted previously addressed key concepts.

This really helped students in remembering key concepts. They would receive feedback on how well they did and could use this information as they move forward.

INTERMEDIATE EXAMPLE: A TEACHER'S EXPERIENCE

Using the carry-over test strategy

An intermediate classroom was focusing on graphing and understanding the Cartesian plane (four quadrants). After checking the students' unit test results, the teacher noticed that many students struggled with the following question.

In which of the four quadrants do the following points lie?

- A (5, –3)
- B (6, 7)
- C (–4, 9)
- D (–1, –1)
- E (–6, 7)
- F (8, 6)
- G (–5, –7)
- H (3, –6)

The teacher applied the carry-over test approach to focus students' minds on the Cartesian plane concept, which so many students had struggled with. Although the focus of the next unit would be operations including decimals, the teacher tacked the following questions onto that unit's test.

Identify if the following statements are true or false.

- The coordinate (–4, 2) lies in quadrant 3.
- The coordinate (5, 1) lies in quadrant 1.
- The coordinate (–3, –6) lies in quadrant 4.
- The coordinate (7, –5) lies in quadrant 2.

When the test was checked, the teacher noticed that considerably fewer students struggling with the Cartesian plane question.

Depending on the results, the teacher might decide to add further review or add another Cartesian plane question onto the next unit test to ensure students have maintained this learning. The teacher would only add one or two questions addressing previous concepts to successive tests so as not to overwhelm students during the testing process.

Correlation to Ontario Ministry of Education's supporting document *High-Impact Instructional Practices in Mathematics*

Tools and Representations

The use of tools and representations supports a conceptual understanding of mathematics, and provides a way for students to think through problems and then communicate their thinking. Tools and representations explicitly and visually represent math ideas that are abstract. When paired with discussion, they help to demonstrate concepts and thinking.

Direct Instruction

Direct instruction is a concise, intentional method of instruction. Learning goals are clearly communicated, models and representations are introduced in context, and questioning and brief activities are incorporated. Common during direct instruction is a think aloud that defines and uses math vocabulary, and makes key concepts and connections explicit. Direct instruction checks for understanding, summarizes the experience, and provides feedback.

STRAND F

Financial Literacy

Strand A	Strand B	Strand C	Strand D	Strand E	**Strand F**
Social-Emotional Learning Skills in Mathematics and Mathematical Processes	Number	Algebra	Data	Spatial Sense	**Financial Literacy**

Financial Literacy is a dedicated strand throughout the elementary math curriculum. Financial literacy is more than just knowing about money and financial matters and having the skills to work with this knowledge. Students develop the confidence and capacity to successfully apply the necessary knowledge, concepts, and skills in a range of relevant real-life contexts and for a range of purposes. They also develop the ability to make informed decisions as consumers and citizens while considering the ethical, societal, environmental, and personal aspects of those decisions.

In Grades 1 to 3, students demonstrate an understanding of the value and use of money by recognizing Canadian coins and bills, representing various amounts, and calculating change in simple transactions. In Grades 4 to 8, students extend their learning to the knowledge, concepts, and skills required to make informed financial decisions relevant to their lived experiences and plan simple sample budgets. Students begin to develop consumer and civic awareness in the junior and intermediate grades.

This strand connects with other mathematics strands in many ways, such as applying knowledge, concepts, and skills related to:

- numbers and operations to calculate change;
- percent to calculate sales tax and interest;
- mathematical modelling to understand real-life financial situations, including the financial applications of linear rates;
- unit rates to compare goods and services, and mental math to quickly determine those with the best value; and
- social-emotional learning to become confident and critical consumers, and to persevere in managing financial well-being.

Key Changes in the Revised Mathematics Curriculum (2020)

No expectations related to financial management in previous math curriculum (2005).
No expectations related to consumer and civic awareness in previous math curriculum (2005).

Learning Strategy Category

Learning By Figuring It Out: Helping students approach problems without first having a defined "correct" strategy

- Use mistakes to learn
- Surf problem-solving strategies
- Predict and check answers to a problem
- Work backward from a given solution

The following are examples of primary, elementary, and intermediate students referencing problems with currency, Canadian coins, and working backward:

PRIMARY EXAMPLE: A STUDENT EXPLAINS

Illusion of mastery of word problems

We spent a lot of time on word problems in the fall. I mean a lot of time. We would add or subtract numbers and then be given word problems to do. Some days, the teacher would give us, like, five or ten addition problems and about ten minutes to solve them. We would then check them as a group. Then the teacher would give us, like, five word problems to solve. I always got mine right. Then other days we would do the same, only subtract.

But it's not fair what happened last week. We were working on finding the perimeter of shapes. Then, on Friday, the teacher gave us three word problems to solve. All the problems were about subtraction and money. I thought I had them all right, but when the teacher went over the answers with everyone, I had them all wrong! I was confused. We were working on perimeter all week—how was I supposed to know that the problems were about something else?! I thought that I knew how to find the answers for word problems, but I guess I don't.

ELEMENTARY EXAMPLE: STUDENT WORK

Remembering the penny to reinforce rounding

To better remember how rounding works, a student thought of how the penny was no longer used in Canada and that cents had to be rounded to the nearest 0, 5, or 10. This concrete example helped to provide context and to serve as a reminder of how rounding works, especially when using decimals.

INTERMEDIATE EXAMPLE: STUDENT WORK

Thinking of a solution, and then working backward to find the right strategy

An intermediate student, when provided a problem, would think of possible solutions that made sense. He then selected a strategy he thought would work. This backward-thinking approach helped him narrow down the list of strategies to try.

Instructional Strategy Category

Rethinking Homework: Reimagining homework as an opportunity for review:

- Assigning homework that supports recall of previous learnings

The following are examples of how primary, elementary, and intermediate teachers crafted a homework program:

PRIMARY EXAMPLE: A TEACHER'S EXPERIENCE

Planning a weekly homework assignment

The teacher assigned the homework on Monday and required students to have it completed by Friday. The homework always consisted of two parts: Part 1 addressed key concepts covered earlier in the year, and Part 2 addressed concepts that were currently being explored. Such an approach to homework reflected a spaced and mixed approach to instruction and learning. On Friday morning, the teacher would check the homework during the first part of the math lesson.

Part 1

- Write the value of each digit in 781.
- Draw base-ten blocks to show 563 in two different ways. You can use a square for a flat, a stick for a rod, and a dot for a unit.

Part 2

- Solve: 674 + 193
- Solve: 301 – 137
- Solve: 902 – 683
- Solve: 339 + 428

ELEMENTARY EXAMPLE: A TEACHER'S EXPERIENCE

Planning a nightly homework assignment

A teacher assigned four problems for homework nightly. The problems dealt with concepts that had been addressed earlier in the year. This strategy was an attempt by the teacher to continuously space and mix concepts to support student learning. She began every math class by posting the solutions to the problems on the board. Students would check their own work. This checking of homework took only about five minutes.

- Solve: 64 x 52
- Solve: 452 ÷ 6
- Place >, <, or = in the circle: $\frac{3}{5} \bigcirc \frac{3}{7}$.
- Write the value of each digit in 18.098.

INTERMEDIATE EXAMPLE: A TEACHER'S EXPERIENCE

Planning a nightly homework assignment

The teacher assigned homework nightly. The first part of the homework addressed the concept being explored in class currently. A second part addressed concepts from earlier in the year. Finally, a third part consisted of problems requiring a little more concentrated effort. The teacher designed this plan to provide mixed and spaced retrieval opportunities that would make learning effortful. The teacher posted solutions on the classroom whiteboard the next morning.

Part 1
- Solve: 4b + 7 = 35
- Solve: 8a – 3 = 53

Part 2
- Solve: (–2) + (–5)
- Solve: (+6) + (–4)
- Solve: (–3) – (–7)
- Solve: (+8) – (+9)

Part 3
- Create a table of values for the expression 3y + 2.
- Draw a graph for the above expression.

Correlation to Ontario Ministry of Education's supporting document *High-Impact Instructional Practices in Mathematics*

Flexible Groupings

Flexible groupings provide opportunities for students to participate in rich mathematical conversations, learn from one another, and strengthen their mathematical understanding. Creating flexible groupings in a math class enables students to work independently of the teacher, but with the support of their peers. Flexible collaborative groups work best because students need to be grouped differently for different reasons.

Conclusion

As can be observed throughout this companion guide, *Making Math Stick: Classroom Strategies that Support Long-Term Understanding of Math Concepts* closely aligns with Ontario's revised Grades 1–8 mathematics curriculum (2020). The few instructional and learning strategies taken from *Making Math Stick* highlight how each strand can be supported in a meaningful and effective way. The strategies contained within *Making Math Stick* not only support student understanding of mathematical concepts but will also enable students to be able to recall previous learnings to solve novel situations. *Making Math Stick* addresses the collective problem of students consistently forgetting concepts and strategies both within the year and from year-to-year.

In addition to aligning with the Ontario's revised Grades 1–8 mathematics curriculum (2020), the approach taken within *Making Math Stick* and the instructional and learning strategies shared coincide with the Ontario Ministry of Education's supporting document *High-Impact Instructional Practices in Mathematics*.

In conclusion, *Making Math Stick* is a professional learning resource that will enable the teacher to implement Ontario's revised Grades 1–8 mathematics curriculum (2020) while addressing instructional practices listed in the Ontario Ministry of Education's supporting document *High-Impact Instructional Practices in Mathematics*. It supports teachers in moving beyond "covering curriculum" to focusing on student learning when planning instruction.

For further support, please reach out to David Costello to discuss how *Making Math Stick* can be a resource to support you and your students:

Costello Math Website (www.costellomath.com)
Email (david@costellomath.com)
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References

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Making Math Stick shows teachers how to foster classroom learning experiences that will help students to better understand, retain, and apply key math concepts. The book explores why we need a different approach to teaching math—one that leads students to a deeper understanding of mathematical concepts that will stick with them long after the test is handed in.

Based on extensive classroom experience, this remarkable book helps teachers stop working harder and start working smarter. It describes a shift from "teach-test-move-on" to "teach-connect-apply" to optimize student learning.

Teachers will find useful insights for adjusting common classroom tools so they can

- strengthen math instruction and take students beyond exams to deeper learning
- engage in instruction that helps students recall and apply math concepts
- focus on addressing all learning needs in K–8 classrooms

In this valuable resource, teachers will find simple, manageable, and sustainable strategies that students can use to build retention. From ideas on learning through self-assessment and building a network of memory, the book goes on to offer suggestions for figuring it out, picturing it, and writing about it. Teachers will find useful instructional strategies they can use to expand the use of formative assessment and rethink testing, reviewing, homework, and the year's plan.

Ideal for new and experienced teachers, this timely book frames instruction so that students strengthen their understanding and can remember and apply learning in novel situations down the road.

Making Math Stick is a game-changer that champions durable learning for all students.

Dr. David Costello is an author and professional learning facilitator who focuses on mathematics instruction and learning. A classroom teacher and administrator based in Prince Edward Island, David has supported teachers in many roles including as a numeracy interventionist, numeracy coach, numeracy leader, and curriculum consultant. A popular speaker at conferences, David has also instructed numerous university courses on curriculum, differentiation, mathematics, and literacy. David is committed to transforming math instruction and creating more meaningful learning experiences for teachers and students.

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